Homework for Chapter 1

1.1. Install software that allows you to record and analyze sound. I recommend Audacity for a computer, TwistedWave for iPhone, and MixPad for Android, all of which are free.[1] Make two five-second recordings of "found sounds" that are not intended to be music. One recording should be a sound that you perceive as highly musical and the other should be something that doesn't sound like music to you.

(a) Identify and describe the sounds that you recorded. Why did you choose them?

(b) Comment on the differences between the pattern (or lack of pattern) in their waveforms, both zoomed out and zoomed in.

(c) Extra Credit: If you don't have one already, create an account at SoundCloud.com. Upload your sounds and add the hashtag #thesoundofnumbers. Send me email at `rhall@sju.edu` with links to your sounds. Please identify yourself in the email so I know who you are.

(d) Extra Credit: Audacity lets you copy, cut, and paste sounds. You can also take a sound and transform it into a different sound using Effects. Incorporate your found sounds into a short composition. Upload your composition as described above.

1.2. Use a counterexample to prove that each of these statements is false.

(a) If n is an integer, then $n \times 1/n = 1$.

(b) All musical compositions involve musicians making sound.

(c) Every musical composition has a specific beginning and end.

1.3. Definition: An integer a is *divisible* by an integer b if $a = bk$, where k is an integer.

Explain why the following is NOT a proof of the statement "The square of any even number is divisible by 4."

Proof. A number is even if it equals $2n$, where n is an integer. Suppose the number is 6, which is even because $6 = 2 \cdot 3$. Then $6^2 = 36$, which is divisible by 4 because $36 = 4 \cdot 9$. The square of any even number is divisible by 4 for the same reason. □

Homework for Chapter 2

2.1. Consider the collection of poetic meters with seven syllables, which may be long or short. How many total meters are there? ________

[1] Audacity is installed on all the computers in the Digital Media Zone, which is located on the second floor of the Post Learning Commons side of the library. If you would like help installing Audacity on your own computer, go to Science Center 129.

How many of these meters have

(a) no short syllables?

(b) one short syllable?

(c) two short syllables?

(d) three short syllables?

(e) four short syllables?

(f) five short syllables?

(g) six short syllables?

(h) seven short syllables?

Hint: this question is extremely tedious if you try to write down all the meters and count them. The best way to answer it is to use the relationship between poetic meters and the *meruprastāra* (Pascal's Triangle).

2.2. Suppose you flip a coin seven times and write down the sequence of heads (H) and tails (T). There is a exact correspondence between sequences of heads and tails and poetic meters: just replace H with L and T with S. How many possible patterns are there? ________

Use your answers to the previous question to determine the percentage of the patterns, rounding to two decimal places, that have

(a) no tails

(b) one tail

(c) two tails

(d) three tails

(e) four tails

(f) five tails

(g) six tails

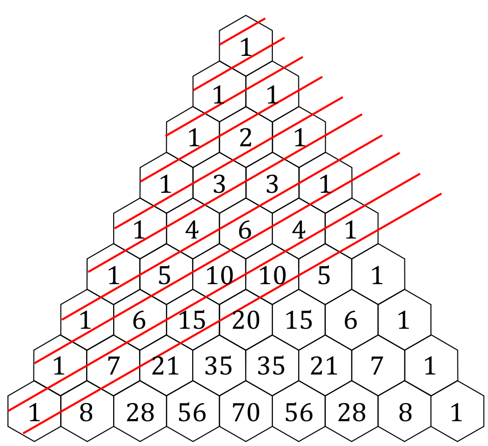
(h) seven tails

Assuming your coin is fair, each of the patterns of heads and tails is equally likely. The percentages you have computed equal the *probabilities* that you will get each number of tails when you flip the coin seven times.

2.3. How many meters have the same duration as SSSSSS (including SSSSSS)? How have the same duration as LLLLLL? Your answers should be Hemachandra numbers.

2.4. Suppose a drummer wants to take a solo that is eight beats long and made up of 1-beat notes, 2-beat notes, and 4-beat notes. How many patterns are possible? Hint: I don't recommend trying to list the patterns, because there are a lot. Rather, find base cases and a recursive rule, write the sequence of number of patterns, and find the eighth number in that sequence. *If you're a musician, the question is, "How many ways can you fill two measures in 4/4 time with quarter notes, half notes, and whole notes?"*

2.5. Add the numbers between the diagonal lines in Pascal's Triangle. For example, the first few sums are 1, 1, 1+1=2, 1+2=3, and 1+3+1=5. What's the pattern?



Solutions (Chapters 1 & 2)

1.1. Answers vary. Your choices of sounds should show that you know the definition of a found sound (that is, a sound that is not intended to be music by whoever or whatever created it). Sounds that seem more musical tend to be more structured: either the overall shape of the waveform has a pattern that indicates the sound is rhythmic, or the waveform has a repetitive pattern when you zoom in; however, this observation isn't a hard and fast rule.

1.2. You have to show a counterexample to each of the statements: (a) 0 is an integer, but $0 \times \frac{1}{0}$ is undefined because $\frac{1}{0}$ is not a number. (b) Answers vary. Any composition where no musician is making a sound is a counterexample. Computer music, found sounds, and Cage's 4'4" are some of the possible answers. (c) Answers vary. Some types of music don't have beginnings and endings. African drumming, algorithmic music, and jam band music are some of the possible answers.

1.3. This is not a proof because it is only an example showing that the square of 6 is divisible by 4. There are an infinite number of examples, so you can't prove the statement by working out examples. Instead, you have to make a logical argument that applies to all even numbers.

2.1. There are $2^7 = 128$ meters. The numbers of meters of each type are found in row 7 of the meruprastāra: (a) 1, (b) 7, (c) 21, (d) 35, (e) 35, (f) 21, (g) 7, (h) 1.

2.2. There are $2^7 = 128$ patterns. The percentages of patterns of each type are found by dividing row 7 of the meruprastāra by 128: (a) 0.78%, (b) 5.47%, (c) 16.41%, (d) 27.34%, (e) 27.34%, (f) 16.41%, (g) 5.47%, (h) 0.78%.

2.3. SSSSSS has duration 1+1+1+1+1+1=6, so the answer is the 6th Hemachandra number: 13. LLLLLL has duration 12 so the answer is the 12th Hemachandra number: 233.

2.4. The technique is to follow the procedure used in the worksheet on page A–3. There are
1 pattern with duration 1 beat: 1
2 patterns with duration 2 beats: 1+1, 2
3 patterns with duration 3 beats: 1+2, 1+1+1, 2+1
6 patterns with duration 4 beats: 4, 1+1+2, 2+2, 1+2+1, 1+1+1+1, 2+1+1
This is the base case. The recursive rule is that each number in the sequence is the sum of the numbers that are one, two, and four places before it. In mathematical notation, if $D[n]$ is the number of patterns with n beats, then $D[n] = D[n-1] + D[n-2] + D[n-4]$. The proof of this is similar to the proof of Theorem 2.2. Using the recursive rule, the sequence is $1, 2, 3, 6, 10, 18, 31, 55, \ldots$ so the answer is 55, the eighth number.

2.5. The answers are Hemachandra/Fibonacci numbers: 1, 1, 2, 3, 5, 8, ...

Homework for Chapter 3, Part 1 (due September 26)

Directions: Write your homework neatly on the front side of the paper. Please do not write on the back. Staple your papers together and cut off any messy edges.

3.1. Draw six beat class circles and notate the patterns

`x.x...    .x.x..    ..x.x.    ...x.x    x...x.    .x...x`

on the circles. How are the pictures related?

3.2. Suppose a piece of music has 12 beats per measure. Assume this is true for all parts of this problem.

(a) Which beat class is 26 beats after the beginning of a measure?

(b) Which beat class is 36 beats after the beginning of a measure?

(c) What is the beat class at time $t100$?

(d) What is the beat class at time $t120$?

(e) Do time points $t45$ and $t165$ belong to the same beat class? Show work.

(f) Do time points $t45$ and $t265$ belong to the same beat class? Show work.

(g) Suppose A and B are time points and $A - B = 36$. Do A and B belong to the same beat class? How do you know?

3.3. Suppose A is a positive integer. Explain why $A \bmod 10$ equals the last digit of A.

3.4. Suppose there are eight beats per measure and you clap every sixth beat, starting with bc0. List the beat classes that you clap on, in the order you clap them. Do you clap on all the possible beat classes?

3.5. Determine whether the following are true or false. Show work.

(a) T/F: $357 \equiv 379 \pmod{11}$

(b) T/F: $30 \equiv 44 \pmod{4}$

3.6. Suppose N is a positive whole number and $0 \equiv 8 \pmod{N}$. What are the possible values of N? (Hint: there are four possible values.)

3.7. List four numbers that are congruent to $0 \pmod{2}$ and four numbers that are congruent to $1 \pmod{2}$. What is the common name for the numbers that are congruent to $0 \pmod{2}$? The numbers that are congruent to $1 \pmod{2}$?

Solutions to Chapter 3 Homework

3.1. You should draw six "six-hour clocks" with the number 0 on top. Each circle has two blobs. The circles are related to each other by $60°$ rotations.

3.2.

(a) bc 4 because $26 \div 12 = 2\ r.4$.

(b) bc 0 because $36 \div 12 = 3$ with no remainder.

(c) Since $100 \div 12 = 8\ r.4$, the beat class is bc 4.

(d) Since $120 \div 12 = 10\ r.0$, the beat class is bc 0.

(e) Yes, since $(45 - 165)/12$ is an integer.

(f) No, since $(45 - 265)/12$ is not an integer.

(g) Yes, because they are separated by three measures (alternately, because $(A - B)/n = 36/12 = 3$ is an integer).

3.3. Since every positive integer A can be written as the sum of a multiple of 10 and the number's last digit, which is less than 10, the last digit is the remainder when A is divided by 10, and, by definition, the remainder equals A mod 10. For example 278 mod $10 = 8$ because $278 = 270 + 8 = 27 \cdot 10 + 8$, so $278 \div 10 = 27\ r.8$.

3.4. The times you clap on are 0, 6, 12, 18, 24, 30, ... Reduce them mod 8 to get the beat classes: bc0, bc6, bc4, bc2, bc0, bc6, ... You do not clap on all possible beat classes—just the even ones.

3.5.

(a) True, because $(357 - 379)/11$ is an integer.

(b) False, because $(30 - 44)/4$ is not an integer.

3.6. Any divisors of 8 work, because you need $(8 - 0)/N$ to be an integer. The possible values are 1, 2, 4, and 8.

3.7. If $a \equiv 0 \pmod 2$, then $(a - 0)/2$ is an integer, so a is divisible by 2. Examples include 2, 4, 6, 8, ... If $a \equiv 1 \pmod 2$, then $(a - 1)/2$ is an integer, so $a - 1$ is divisible by 2. Examples include 1, 3, 5, 7, ... They are the even numbers and the odd numbers.
4 points